National Qualifications 2024

X847/77/11

Mathematics
Paper 1 (Non-calculator)

MONDAY, 13 MAY
9:00 AM – 10:00 AM

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

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FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. Differentiate the following with respect to x:

(a) $y = \cot 3x$ 2

(b) $f(x) = 5x(4x-7)^{\frac{1}{2}}$. 2

2. A complex number is defined by $z = 1+i$.

(a) Express z in polar form. 2

(b) Use de Moivre's theorem to evaluate z^8. 2

3. A geometric sequence of positive terms has third term 36 and fifth term 16.

(a) Calculate the value of the common ratio. 2

(b) Calculate the value of the first term. 1

(c) State why the associated geometric series has a sum to infinity. 1

(d) Find the value of this sum to infinity. 1

4. Matrix A is defined by $A = \begin{pmatrix} 6 & 1 \\ 11 & 3 \end{pmatrix}$.

(a) Find A^{-1}, the inverse of matrix A. 2

Matrix B is defined by $B = \begin{pmatrix} -4 & 3 \\ -5 & 2 \end{pmatrix}$.

(b) Find the matrix M such that $AM = B$. 2

MARKS

5. The function $f(x)$ is defined by $f(x) = x^3 - x,\ x \in \mathbb{R}$.

(a) Determine whether $f(x)$ is even, odd or neither. 2

(b) Show that the graph of $y = f(x)$ has a point of inflection. 2

6. (a) Find the 2×2 matrix, A, associated with a reflection in the x-axis. 1

(b) Describe the transformation associated with the matrix $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. 1

(c) Find the 2×2 matrix, C, associated with a reflection in the x-axis followed by the transformation associated with $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. 2

7. A curve is defined by the equation $x^2y + 4xy^2 = -32,\ y > 0$.

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$. 3

The curve has only one stationary point.

(b) Find the coordinates of the stationary point. 3

8. Use the substitution $u = \tan 2x$ to evaluate $\int_0^{\frac{\pi}{8}} \frac{\sqrt{\tan 2x}}{\cos^2 2x}\,dx$. 4

[END OF QUESTION PAPER]