National
Qualifications
2018

2018 Statistics

Advanced Higher

Finalised Marking Instructions

General marking principles for Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

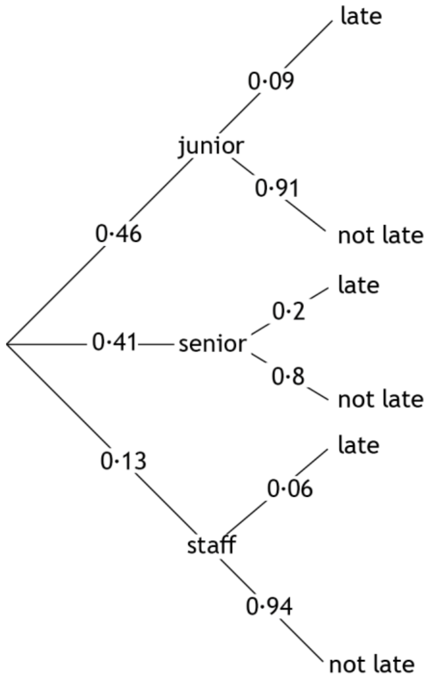
In this case, award 3 marks.

Detailed marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] correct distribution •[2] calculate probability	•[1] $X \sim \text{Po}(2)$ •[2] $\text{P}(X = 3) = 0{\cdot}1804$	**2**
			Calculate the probability that, during a given night, neither baby wakes up.		
	(b)		•[3] correct distribution •[4] calculate probability	•[3] $Y \sim \text{Po}(5)$ •[4] $\text{P}(Y = 0) = 0{\cdot}0067$	**2**

Notes:

An alternative for (b): $\text{Po}(2,\,0)\ \text{Po}(3,\,0) = 0{\cdot}1353 \times 0{\cdot}0498 = 0{\cdot}0067$

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)	(i)	•[1] correct probability	•[1] 0·4186	**3**
		(ii)	•[2] appropriate strategy •[3] calculate probability	•[2]  •[3] 0·1312	
Notes: Other methods are acceptable					
	(b)		•[4] appropriate strategy •[5] calculate probability	•[4] $\mathrm{P}(S\mid L)=\dfrac{\mathrm{P}(S\cap L)}{P(L)}$ •[5] $\dfrac{0{\cdot}41\times 0{\cdot}2}{0{\cdot}1312}=0{\cdot}625$	**2**
Notes: Other methods are acceptable					
	(c)		•[6] appropriate strategy •[7] appropriate description	•[6] randomly sample •[7] 10% of juniors, 10% of seniors and 10% of staff	**2**
Notes:					

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] appropriate hypotheses •[2] correct b •[3] correct s^2 •[4] calculate t •[5] correct critical value •[6] deal with H_0 •[7] appropriate conclusion	•[1] $H_0: \beta = 0 \; H_1: \beta \neq 0$ •[2] $b = \frac{S_{xy}}{S_{xx}} = 0{\cdot}22642$ •[3] $s^2 = \frac{SSR}{n-2} = 22{\cdot}098$ •[4] $t = \frac{b\sqrt{S_{xx}}}{s} = 4{\cdot}82$ •[5] the 5% cv is 2·571 •[6] 4·82 > 2·571 so we reject H_0 at the 5% level of significance and •[7] conclude that there is evidence that the slope parameter is non-zero	**7**
	(b)		•[8] appropriate comment •[9] appropriate reason	•[8] the coefficient of determination •[9] high values would make it useful for prediction	**2**

Notes:

full credit should be given for the knowledge that first finding r and then using

$t = \frac{r\sqrt{n-2)}}{\sqrt{1-r^2}}$ yields the same value of t

•[5] the alternative p-value approach (PvA) would record that $2P(t_5 > 4{\cdot}82) = 0{\cdot}0048 < 0{\cdot}025$ etc

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] appropriate strategy •[2] calculate probability	$P(X > 10\,100)$ •[1] $= P\left(Z > \frac{10\,100 - 10\,000}{250}\right)$ •[2] $0{\cdot}3446$	**2**
Notes:					
	(b)		•[3] appropriate strategy •[4] correct z-value •[5] calculate life expectancy	$P(X > x)$ •[3] $= P\left(Z > \frac{x - 10\,000}{250}\right)$ $= 0{\cdot}9$ •[4] $z = -1{\cdot}28$ •[5] 9680 hours	**3**
	(c)		•[6] appropriate assumption •[7] combine random variables •[8] correct μ •[9] correct σ^2 •[10] appropriate strategy •[11] calculate probability	•[6] assuming all weights are independent $W = (B_1 + B_2 + \ldots B_{100}) +$ •[7] $(X_1 + X_2 + \ldots X_{100}) + C$ •[8] $E(W) = 2975$ •[9] $V(W) = 157$ $P(W < 3000)$ •[10] $= P\left(Z < \frac{3000 - 2975}{\sqrt{157}}\right)$ •[11] $= 0{\cdot}9772$	**6**
Notes: other appropriate assumptions may be acceptable					

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] appropriate method	•[1] systematic sampling	**1**
	(b)		•[2] appropriate reason	•[2] The distribution of pebble sizes may not be random, so a single sample from one point might have pebbles that are in some way unrepresentative of the stream bed eg all larger than average.	**1**
Notes:					
	(c)		•[3] correct distribution •[4] correct critical values •[5] appropriate strategy •[6] calculate interval •[7] appropriate interpretation	•[3] $\bar{X} \approx \mathrm{N}\left(119{\cdot}4, \frac{21{\cdot}6^2}{100}\right)$ •[4] $\pm 1{\cdot}64$ •[5] $\bar{X} \pm 1{\cdot}64.\frac{21{\cdot}6}{10}$ •[6] $(115{\cdot}86, 122{\cdot}94)$ •[7] 115·3 is outwith the confidence interval, furnishing evidence to suggest that the estimate of pebble size under the new scheme is significantly different from that of the old.	**5**
Notes:					

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ correct number of arrangements •² appropriate strategy •³ any 6 arrangements •⁴ other 6 arrangements •⁵ appropriate completion	•¹ ${}^{11}C_4 = 330$ •² a set of 4 ranks which sum to at most 14: •³ eg 1234 1235 1236 1237 1238 1256 •⁴ eg 1245 1246 1247 1345 1346 2345 •⁵ giving a total of 12 out of 330	5
Notes:					

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] correct hypotheses	•[1] H_0: the data fit a B(6, 0·1) H_1: it does not	7
			•[2] calculate expected fr's	•[2] 63·8 42·5 11·8 1·7 0·1 0 0	
			•[3] deal with small E_i	•[3] combining to 63·8 42·5 13·6	
			•[4] calculate x^2	•[4] $x^2 = 7·335$	
			•[5] correct critical value	•[5] $\chi^2_{2,0·95} = 5·991$	
			•[6] deal with H_0	•[6] $7·335 > 5·991$ so we reject H_0 at the 5% level of significance	
			•[7] appropriate conclusion	•[7] and conclude that there is evidence against the claim	
Notes: the PvA would record that $P(\chi^2_2 > 7·335) = 0·0255 < 0·05$					
	(b)		•[8] correct observation	•[8] the failure of bulbs may not be independent	1
Notes: For •[8] an alternative would be that it is not realistic to assume that the probability of failure is constant					

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•1 state hypotheses	•1 H_0: $p = 0 \cdot 119$ H_1: $p > 0 \cdot 119$	**6**
			•2 appropriate strategy	•2 $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$	
			•3 correct z value	•3 $= \dfrac{0 \cdot 18 - 0 \cdot 119}{\sqrt{\dfrac{0 \cdot 119 \times 0 \cdot 881}{100}}} = 1 \cdot 88$	
			•4 correct critical value	•4 5% cv is $1 \cdot 64$	
			•5 deal with H_0	•5 $1 \cdot 88 > 1 \cdot 64$ so we reject H_0 at the 5% level of significance	
			•6 appropriate comment	•6 and conclude that there is evidence of the proportion of accidents at this location in 2013 being greater than the 2008–2012 national figure	

Notes:

The alternative use of the normal approximation to the binomial distribution with continuity correction and a p-value of $0 \cdot 0418$ (or $z = 1 \cdot 73$) is acceptable

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•7 critical value	•7 $-1 \cdot 64 =$	**3**
			•8 form equation	•8 $= \dfrac{\dfrac{d}{40} - 0 \cdot 119}{\sqrt{\dfrac{0 \cdot 119 \times 0 \cdot 881}{40}}}$	
			•9 solve and interpret equation	•9 $d = 1 \cdot 4$, so maximum number of drivers is 1	

Notes:
The use of trial and improvement with the binomial distribution is acceptable.

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] correct target value •[2] correct substitution •[3] calculate limits	•[1] $\overline{x}=\frac{54{\cdot}3}{10}=5{\cdot}43$ •[2] 1σ limits are $5{\cdot}43\pm\sqrt{\frac{0{\cdot}0576}{5}}$ •[3] 5·323, 5·537	**3**
Notes:					
	(b)		•[4] correct strategy •[5] correct total •[6] calculate minimum pH	•[4] out of control if 21st batch mean is above 1-sigma limit. WECO 4/5 above 1-sigma •[5] $5{\cdot}537\times 5=27{\cdot}685$ •[6] total of 4 values $=21{\cdot}942$ $27{\cdot}685-21{\cdot}942=5{\cdot}743$	**3**
Notes:					

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)	(i)			3

X	7	8	9	10	11	12	13	14	15	16	17
$\mathrm{P}(X=x)$	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{5}{30}$	$\frac{4}{30}$	$\frac{2}{30}$	$\frac{0}{30}$	$\frac{2}{30}$

Question			Generic scheme	Illustrative scheme	Max mark
			•[1] correct distribution	•[1] See table above	
		(ii)	•[2] correct strategy	•[2] $\mathrm{V}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$	
			•[3] correct substitution	•[3] $\mathrm{V}(X)=150-12^2=6$	

Notes:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•[4] correct distribution	•[4] $X \approx \mathrm{N}(12,6)$	6
			•[5] appropriate reason	•[5] the shape of the graph indicates a normal distribution	
		(ii)	•[6] correct continuity corrections	•[6] $\mathrm{P}(X=10)=\mathrm{P}(9{\cdot}5<X<10{\cdot}5)$	
			•[7] calculate z-values	•[7] $=\mathrm{P}\left(\frac{9{\cdot}5-12}{\sqrt{6}}<Z<\frac{10{\cdot}5-12}{\sqrt{6}}\right)$ $=\mathrm{P}(-1{\cdot}02<Z<-0{\cdot}61)$	
			•[8] calculate p-value	•[8] $=0{\cdot}1164$	
		(iii)	•[9] appropriate suggestion	•[9] increase the number of games played	

Notes:
An alternative •[9] would be to perform a chi-squared test

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(i)	•[1] correct country	•[1] Czech Republic	**3**
		(ii)	•[2] appropriate reason	•[2] Although there appears to be positive correlation this in no way indicates causation	
		(iii)	•[3] appropriate feature	•[3] The variance of the data points around the fitted line is not constant	

Notes:

Alternative to •[2]: We would need to see a negative correlation before looking for evidence of an association between increased welfare and encouraging people not to work

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] correct S_{xy}	•[4] $S_{xy} = 141677\cdot3 - \frac{2208 \times 1054\cdot8}{18}$ $= 12288\cdot5$	**6**
			•[5] calculate b	•[5] $b = \frac{12288\cdot5}{105904} = 0\cdot1160$	
			•[6] calculate a	•[6] $a = \frac{1054\cdot8}{18} - 0\cdot1160 \times \frac{2208}{18}$ $= 44\cdot37$	
			•[7] state equation	•[7] $y = 44\cdot37 + 0\cdot1160\,x$	
			•[8] calculate $\hat{Y}$	•[8] $\hat{Y} = 44\cdot37 + 0\cdot1160 \times 59 = 51\cdot214$	
			•[9] calculate residual	•[9] $\text{residual} = 42\cdot6 - 51\cdot214 = -8\cdot6$	

Notes:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)	(i)	•[1] appropriate hypotheses	•[1] H_0: $\mu_a = \mu_b$ H_1: $\mu_a \neq \mu_b$	**10**
			•[2] correct statistics	•[2] $\bar{x}_b = 14 \cdot 1$ $s_b^2 = 10 \cdot 8$	
			•[3] correct test statistic	•[3] $T_{n_a+n_b-2} = \dfrac{\bar{X}_a - \bar{X}_b}{s\sqrt{\dfrac{1}{n_a} + \dfrac{1}{n_b}}}$	
			•[4] correct s^2	•[4] $s^2 = 5 \cdot 58$	
			•[5] calculate t	•[5] $\lvert t \rvert = 1 \cdot 43$	
			•[6] correct critical value	•[6] 5% cv is $t_{16,0.975} = 2 \cdot 12$	
			•[7] deal with H_0	•[7] $1 \cdot 43 < 2 \cdot 12$ so we cannot reject H_0 at the 5% significance level	
			•[8] appropriate conclusion	•[8] and conclude that there is no evidence of a difference in mean weights	
			•[9&10] appropriate assumptions	•[9&10] we have assumed that the two fish populations' weights are distributed normally with equal variances	
		(ii)	•[11] appropriate comment	•[11] the sample variances are far from being equal	**1**
Notes: the PvA would record that $2P(t_{16}>1 \cdot 43) = 0 \cdot 1720 > 0 \cdot 05$					
	(b)		•[12] correct test statistic	•[12] $Z = \dfrac{\bar{X}_a - \bar{X}_b}{\sigma\sqrt{\dfrac{1}{n_a} + \dfrac{1}{n_b}}}$ where $\sigma = 1 \cdot 5$	**3**
			•[13] correct cv and inequality	•[13] $z = -2 \cdot 25 < -1 \cdot 96$	
			•[14] appropriate conclusion	•[14] and we may conclude that there **is** evidence of different mean weights	
Notes: the PvA would record that $2P(z<-2 \cdot 25) = 0 \cdot 0244$					
	(c)		•[15] correct test	•[15] Mann-Whitney	**2**
			•[16] correct assumption	•[16] population distributions have the same shape and variability	
Notes:					

[END OF MARKING INSTRUCTIONS]